EXPRESSIONS AND EQUATIONS
6TH GRADE

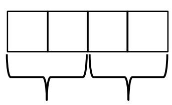
Lesson 1: The Relationship of Addition and Subtraction

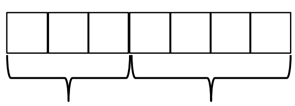
Classwork

Opening Exercise

a. Draw a tape diagram to represent the following expression: $5 + 4$.

b. Write an expression for each tape diagram.

i. 

ii. 

Exercises

Predict what will happen when a tape diagram has a large number of squares, some squares are removed, and then the same amount of squares are added back on.

1. Build a tape diagram with 10 squares.

   a. Remove six squares. Write an expression to represent the tape diagram.

   b. Add six squares onto the tape diagram. Alter the original expression to represent the current tape diagram.

c. Evaluate the expression.

2. Write an equation, using variables, to represent the identities we demonstrated with tape diagrams.

3. Using your knowledge of identities, fill in each of the blanks.

a. $4 + 5 - \_\_\_\_\_ = 4$

b. $25 - \_\_\_\_\_ + 10 = 25$

c. $\_\_\_\_\_ + 16 - 16 = 45$

d. $56 - 20 + 20 = \_\_\_\_\_$

4. Using your knowledge of identities, fill in each of the blanks.

a. $a + b - \_\_\_\_\_ = a$

b. $c - d + d = \_\_\_\_\_$

c. $e + \_\_\_\_\_ - f = e$

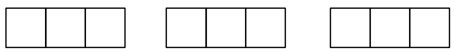
d. $\_\_\_\_\_ - h + h = g$

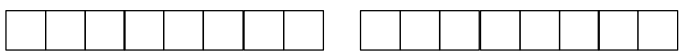
Lesson 3: The Relationship of Multiplication and Addition

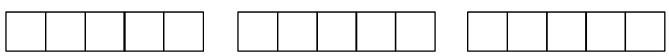
Classwork

Opening Exercise

Write two different expressions that can be depicted by the tape diagram shown. One expression should include addition, while the other should include multiplication.

a. 

b. 

c. 

Exercises

1. Write the addition sentence that describes the model and the multiplication sentence that describes the model.

2. Write an equivalent expression to demonstrate the relationship of multiplication and addition.
   a. $6 + 6$

   b. 3 + 3 + 3 + 3 + 3 + 3

   c. $4 + 4 + 4 + 4 + 4$

   d. 6×2

   e. 4×6

   f. 3×9

   g. $h + h + h + h + h$

   h. $6y$

3. Roberto is not familiar with tape diagrams and believes that he can show the relationship of multiplication and addition on a number line. Help Roberto demonstrate that the expression 3×2 is equivalent to $2 + 2 + 2$ on a number line.

4. Tell whether the following equations are true or false. Then, explain your reasoning.
   a. $x + 6g - 6g = x$

   b. $2f - 4e + 4e = 2f$

5. Write an equivalent expression to demonstrate the relationship between addition and multiplication.
   a. $6 + 6 + 6 + 6 + 4 + 4 + 4$

   b. $d + d + d + w + w + w + w + w$

   c. $a + a + b + b + b + c + c + c + c$

Lesson 5: Exponents

Classwork

Opening Exercise

As you evaluate these expressions, pay attention to how you arrived at your answers.

$4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4$

$9 + 9 + 9 + 9 + 9$

$10 + 10 + 10 + 10 + 10$

Examples 1–5

Write each expression in exponential form.

1. $5 \times 5 \times 5 \times 5 \times 5 =$

2. $2 \times 2 \times 2 \times 2 =$

Write each expression in expanded form.

3. 8^3

4. 10^6

5. g^3

Go back to Examples 1–4, and use a calculator to evaluate the expressions.

What is the difference between $3g$ and g^3?

Examples 6–8

1. Write the expression in expanded form, and then evaluate.

   $3^4 =$

2. Write the expression in exponential form, and then evaluate.

   $4 \times 4 \times 4 \quad =$

3. Write the expression in exponential form, and then evaluate.

   $1.5 \times 1.5 =$

The base number can also be a fraction. Remember how to multiply fractions!

Examples 9–10

4. Write the expression in exponential form, and then evaluate.

   $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} =$

5. Write the expression in expanded form, and then evaluate.

   $\left(\frac{2}{3}\right)^2 =$

Exercises

Fill in the chart, supplying the missing expression.

1. Fill in the missing expression for each row. For whole number and decimal bases, use a calculator to find the standard form of the number. For fraction bases, leave your answer as a fraction.

Exponential Form	Expanded Form	Standard Form
3^2	3×3	9
	$2 \times 2 \times 2 \times 2 \times 2 \times 2$	
4^5		
	$\frac{3}{4} \times \frac{3}{4}$	
	2.1×2.1	

2. Write five cubed in all three forms: exponential form, expanded form, and standard form.

3. Write *three-fifths squared* in all three forms.

4. One student thought two to the third power was equal to six. What mistake do you think he made, and how would you help him fix his mistake?

Lesson 6: The Order of Operations

Classwork

Example 1: Expressions with Only Addition, Subtraction, Multiplication, and Division

What operations are evaluated first?

What operations are always evaluated last?

Exercises 1–3

1. $4 + 2 \times 7$

2. $36 \div 3 \times 4$

3. $20 - 5 \times 2$

Example 2: Expressions with Four Operations and Exponents

$$4 + 9^2 \div 3 \times 2 - 2$$

What operation is evaluated first?

What operations are evaluated next?

What operations are always evaluated last?

What is the final answer?

Exercises 4–5

4. $90 - 5^2 \times 3$

5. $4^3 + 2 \times 8$

Example 3: Expressions with Parentheses

Consider a family of 4 that goes to a soccer game. Tickets are $\$5.00$ each. The mom also buys a soft drink for $\$2.00$. How would you write this expression?

How much will this outing cost?

Consider a different scenario: The same family goes to the game as before, but each of the family members wants a drink. How would you write this expression?

Why would you add the 5 and 2 first?

How much will this outing cost?

How many groups are there?

What is each group comprised of?

Exercises 6–7

6. $2 + (9^2 - 4)$

7. $2 \cdot \left(13 + 5 - 14 \div (3 + 4)\right)$

Example 4: Expressions with Parentheses and Exponents

$$2 \times (3 + 4^2)$$

Which value will we evaluate first within the parentheses? Evaluate.

Evaluate the rest of the expression.

What do you think will happen when the exponent in this expression is outside of the parentheses?

$$2 \times (3 + 4)^2$$

Will the answer be the same?

Which should we evaluate first? Evaluate.

What happens differently here than in our last example?

What should our next step be?

Evaluate to find the final answer.

What do you notice about the two answers?

What was different between the two expressions?

What conclusions can you draw about evaluating expressions with parentheses and exponents?

Exercises 8–9

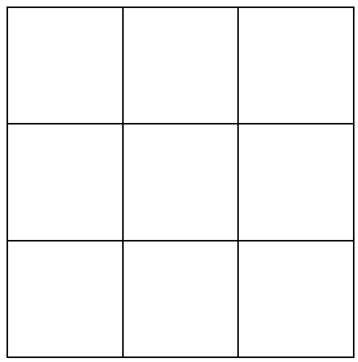
8. $7 + (12 - 3^2)$

9. $7 + (12 - 3)^2$

Lesson 7: Replacing Letters with Numbers

Classwork

Example 1



What is the length of one side of this square?

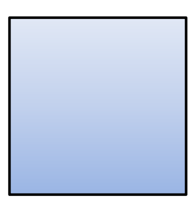
What is the formula for the area of a square?

What is the square's area as a multiplication expression?

What is the square's area?

We can count the units. However, look at this other square. Its side length is 23 cm. That is just too many tiny units to draw. What expression can we build to find this square's area?

What is the area of the square? Use a calculator if you need to.

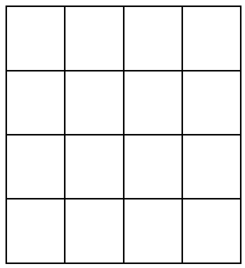


23 cm

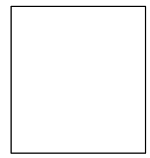
Exercise 1

Complete the table below for both squares. Note: These drawings are not to scale.

$s = 4$

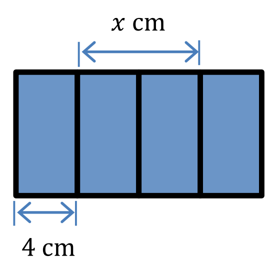


$s = 25$ in.



Length of One Side of the Square	Square's Area Written as an Expression	Square's Area Written as a Number

Example 2



What does the letter b represent in this blue rectangle?

With a partner, answer the following question: Given that the second rectangle is divided into four <u>equal</u> parts, what number does the x represent?

How did you arrive at this answer?

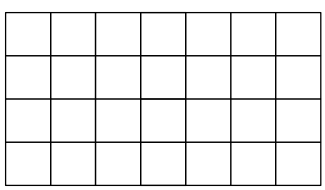
What is the total length of the second rectangle? Tell a partner how you know.

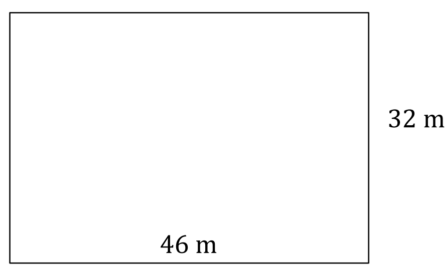
If the two large rectangles have equal lengths and widths, find the area of each rectangle.

Discuss with your partner how the formulas for the area of squares and rectangles can be used to evaluate area for a particular figure.

Exercise 2

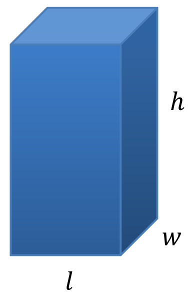
Complete the table below for both rectangles. Note: These drawings are not to scale. Using a calculator is appropriate.

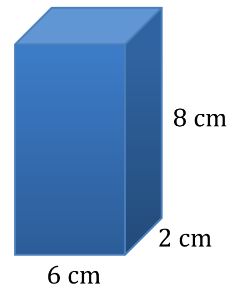




Length of Rectangle	Width of Rectangle	Rectangle's Area Written as an Expression	Rectangle's Area Written as a Number

Example 3





What does the l represent in the first diagram?

What does the w represent in the first diagram?

What does the h represent in the first diagram?

Since we know the formula to find the volume is $V = l \times w \times h$, what number can we substitute for the l in the formula? Why?

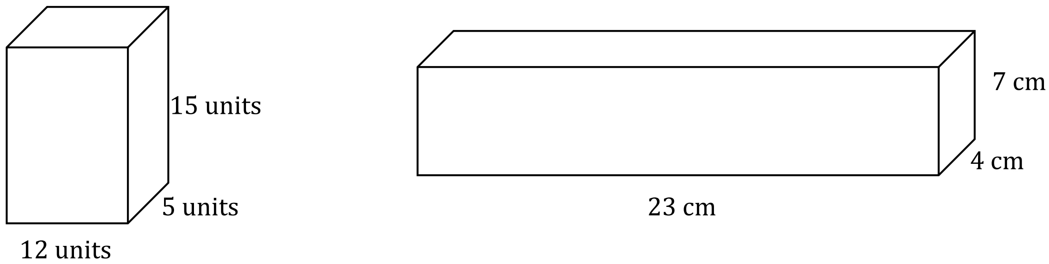
What number can we substitute for the w in the formula? Why?

What number can we substitute for the h in the formula?

Determine the volume of the second right rectangular prism by substituting the letters in the formula with their appropriate numbers.

Exercise 3

Complete the table for both figures. Using a calculator is appropriate.



Length of Rectangular Prism	Width of Rectangular Prism	Height of Rectangular Prism	Rectangular Prism's Volume Written as an Expression	Rectangular Prism's Volume Written as a Number

Lesson 8: Replacing Numbers with Letters

Classwork

Opening Exercise

$$4 + 0 = 4$$
$$4 \times 1 = 4$$
$$4 \div 1 = 4$$
$$4 \times 0 = 0$$
$$1 \div 4 = \frac{1}{4}$$

How many of these statements are true?

How many of those statements would be true if the number 4 was replaced with the number 7 in each of the number sentences?

Would the number sentences be true if we were to replace the number 4 with any other number?

What if we replaced the number 4 with the number 0? Would each of the number sentences be true?

What if we replace the number 4 with a letter g? Please write all 4 expressions below, replacing each 4 with a g.

Are these all true (except for $g = 0$) when dividing?

Example 1: Additive Identity Property of Zero

$$g + 0 = g$$

Remember a letter in a mathematical expression represents a number. Can we replace g with any number?

Choose a value for g, and replace g with that number in the equation. What do you observe?

Will all values of g result in a true number sentence?

Example 2: Multiplicative Identity Property of One

$$g \times 1 = g$$

Remember a letter in a mathematical expression represents a number. Can we replace g with any number?

Choose a value for g, and replace g with that number in the equation. What do you observe?

Example 3: Commutative Property of Addition and Multiplication

$$3 + 4 = 4 + 3$$
$$3 \times 4 = 4 \times 3$$

Replace the 3's in these number sentences with the letter a.

Choose a value for a, and replace a with that number in each of the equations. What do you observe?

Will all values of a result in a true number sentence? Experiment with different values before making your claim.

Now write the equations again, this time replacing the number 4 with a variable, b.

Will all values of a and b result in true number sentences for the first two equations? Experiment with different values before making your claim.

Example 4

$$3 + 3 + 3 + 3 = 4 \times 3$$

$$3 \div 4 = \frac{3}{4}$$

Replace the 3's in these number sentences with the letter a.

Choose a value for a and replace a with that number in each of the equations. What do you observe?

Will all values of a result in a true number sentence? Experiment with different values before making your claim.

Now write the equations again, this time replacing the number 4 with a variable, b.

Will all values of a and b result in true number sentences for the equations? Experiment with different values before making your claim.

Lesson 9: Writing Addition and Subtraction Expressions

Classwork

Example 1

Create a bar diagram to show 3 plus 5.

How would this look if you were asked to show 5 plus 3?

Are these two expressions equivalent?

Example 2

How can we show a number increased by 2?

Can you prove this using a model? If so, draw the model.

Example 3

Write an expression to show the sum of m and k.

Which property can be used in Examples 1–3 to show that both expressions given are equivalent?

Example 4

How can we show 10 minus 6?

- Draw a bar diagram to model this expression.

- What expression would represent this model?

- Could we also use $6 - 10$? Explain.

Example 5

How can we write an expression to show 3 less than a number?

- Start by drawing a diagram to model the subtraction. Are we taking away from the 3 or the unknown number?

- What expression would represent this model?

Example 6

How would we write an expression to show the number c being subtracted from the sum of a and b?

- Start by writing an expression for "the sum of a and b."

- Now show c being subtracted from the sum.

Example 7

Write an expression to show the number c minus the sum of a and b.

Replace the variables with numbers to see if $c - (a + b)$ is the same as $c - a + b$.

Why are the parentheses necessary in this example and not the others?

Exercises

1. Write an expression to show the sum of 7 and 1.5.

2. Draw models to prove that w + 4 and 4 + w represent the same thing.

3. Write an expression to show the sum of a, b, and c.

4. Write an expression and a model showing 3 less than p.

5. Write an expression to show the difference of 3 and p.

6. Write an expression to show 4 less than the sum of g and 5.

7. Write an expression to show 4 decreased by the sum of g and 5.

8. Should Exercises 6 and 7 have different expressions? Why or why not?

Lesson 10: Rewriting Multiplication Expressions

Classwork

Example 1

Write each expression using the fewest number of symbols and characters. Circle the coefficient and underline the variable(s) in your answer.

a. $6 \times b$

b. $4 \cdot 3 \cdot h$

c. $2 \times 2 \times 2 \times a \times b$

d. $5 \times m \times 3 \times p$

e. $1 \times g \times w$

Example 2

a. Find the product of $4f \cdot 7g$.

b. Multiply $3de \cdot 9yz$.

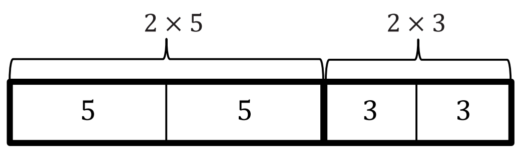
c. Double the product of $6y$ and $3bc$.

Lesson 11: Factoring Expressions

Classwork

Example 1

a. Use the model to answer the following questions.



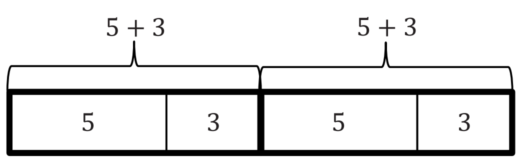
How many fives are in the model?

How many threes are in the model?

What does the expression represent in words?

What expression could we write to represent the model?

b. Use the new model and the previous model to answer the next set of questions.



How many fives are in the model?

How many threes are in the model?

What does the expression represent in words?

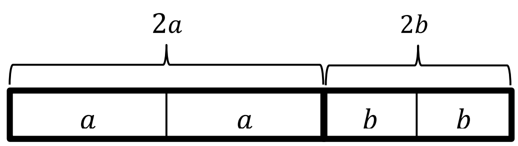
What expression could we write to represent the model?

c. Is the model in part (a) equivalent to the model in part (b)?

d. What relationship do we see happening on either side of the equal sign?

Example 2

1. Now, we will take a look at an example with variables. Discuss the questions with your partner.



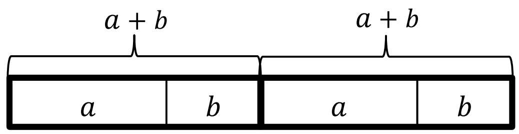
What does the model represent in words?

What does $2a$ mean?

How many a's are in the model?

How many b's are in the model?

2. What expression could we write to represent the model?



How many a's are in the expression?

How many b's are in the expression?

What expression could we write to represent the model?

Are the two expressions #1 and #2 in Example 2 equivalent?

Example 3

Use GCF and the distributive property to write equivalent expressions.

1. $3f + 3g =$ ____________________

   What is the question asking us to do?

   How would Problem 1 look if we expanded each term?

   What is the GCF in Problem 1?

   How can we use the GCF to rewrite this?

2. $6x + 9y =$

   What is the question asking us to do?

   How would Problem 2 look if we expanded each term?

   What is the GCF in Problem 2?

   How can we use the GCF to rewrite this?

3. $3c + 11c =$

   Is there a GCF in Problem 3?

   Rewrite the expression using the distributive property.

4. $24b + 8 =$

   Explain how you used GCF and the distributive property to rewrite the expression in Problem 4.

   Why is there a 1 in the parentheses?

   How is this related to the first two examples?

Exercises

1. Apply the distributive property to write equivalent expressions.

   a. $7x + 7y$

   b. $15g + 20h$

   c. $18m + 42n$

   d. $30a + 39b$

   e. $11f + 15f$

   f. $18h + 13h$

   g. $55m + 11$

   h. $7 + 56y$

2. Evaluate each of the expressions below.

   a. $6x + 21y$ and $3(2x + 7y)$ $x = 3$ and $y = 4$

   b. $5g + 7g$ and $g(5 + 7)$ $g = 6$

c. $14x + 2$ and $2(7x + 1)$ $\quad x = 10$

d. Explain any patterns that you notice in the results to parts (a)–(c).

e. What would happen if other values were given for the variables?

Closing

How can you use your knowledge of GCF and the distributive property to write equivalent expressions?

Find the missing value that makes the two expressions equivalent.

Lesson 12: Distributing Expressions

Classwork

Opening Exercise

a. Create a model to show 2×5.

b. Create a model to show $2 \times b$, or $2b$.

Example 1

Write an expression that is equivalent to $2(a + b)$.

Create a model to represent $(a + b)$.

The expression $2(a + b)$ tells us that we have 2 of the $(a + b)$'s.

Create a model that shows 2 groups of $(a + b)$.

How many a's and how many b's do you see in the diagram?

How would the model look if we grouped together the a's and then grouped together the b's?

What expression could we write to represent the new diagram?

What conclusion can we draw from the models about equivalent expressions?

Let $a = 3$ and $b = 4$.

What happens when we double $(a + b)$?

Example 2

Write an expression that is equivalent to double $(3x + 4y)$.

How can we rewrite double $(3x + 4y)$?

Is this expression in factored form, expanded form, or neither?

Let's start this problem the same way that we started the first example. What should we do?

How can we change the model to show $2(3x + 4y)$?

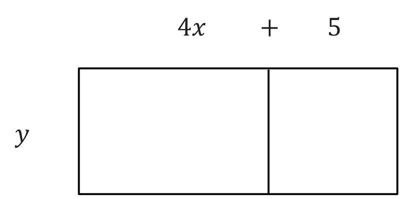
Are there terms that we can combine in this example?

What is an equivalent expression that we can use to represent $2(3x + 4y)$?

Summarize how you would solve this question without the model.

Example 3

Write an expression in expanded form that is equivalent to the model below.



What factored expression is represented in the model?

How can we rewrite this expression in expanded form?

Example 4

Write an expression in expanded form that is equivalent to $3(7d + 4e)$.

Exercises

Create a model for each expression below. Then write another equivalent expression using the distributive property.

1. $3(x + y)$

2. $4(2h + g)$

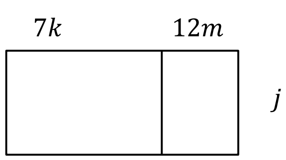
Apply the distributive property to write an equivalent expression in expanded form.

1. $8(h + 3)$

2. $3(2h + 7)$

3. $5(3x + 9y)$

4. $4(11h + 3g)$

5. 

Lesson 13: Writing Division Expressions

Classwork

Example 1

Write an expression showing $1 \div 2$ without the use of the division symbol.

Example 2

Write an expression showing $a \div 2$ without the use of the division symbol.

When we write division expressions using the division symbol we represent ________________.

How would this look when we write division expressions using a fraction?

Example 3

a. Write an expression showing $a \div b$ without the use of the division symbol.

b. Write an expression for g divided by the quantity h plus 3.

c. Write an expression for the quotient of the quantity m reduced by 3 and 5.

Exercises

Write each expression two ways: using the division symbol and as a fraction.

a. 12 divided by 4.

b. 3 divided by 5.

c. a divided by 4.

d. The quotient of 6 and m.

e. Seven divided by the quantity x plus y.

f. y divided by the quantity x minus 11.

g. The sum of the quantity h and 3 divided by 4.

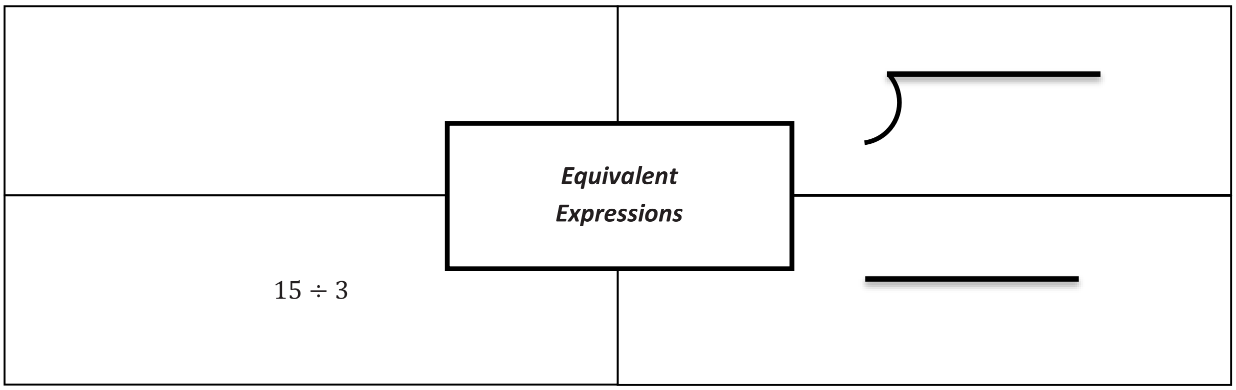
h. The quotient of the quantity k minus 10 and m.

Lesson 14: Writing Division Expressions

Classwork

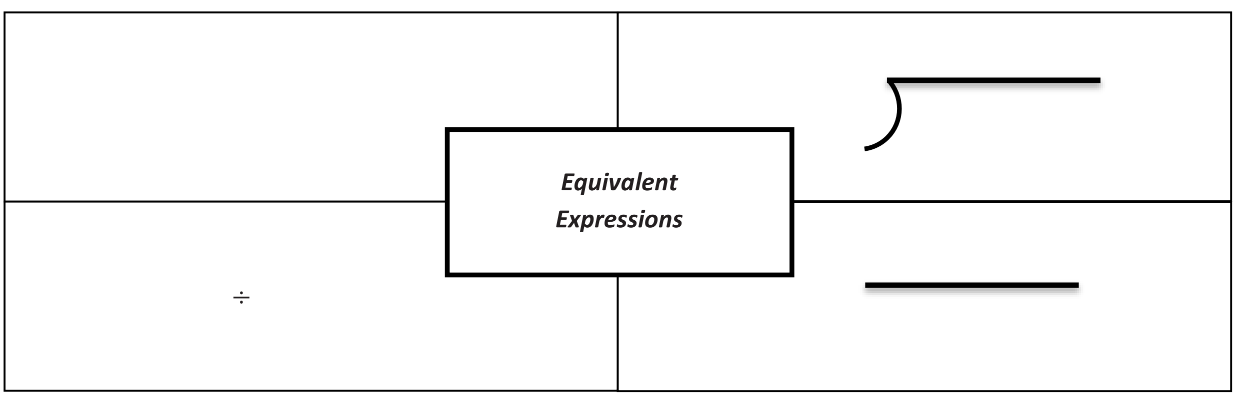
Example 1

Fill in the three remaining squares so that all the squares contain equivalent expressions.



Example 2

Fill in a blank copy of the four boxes using the words *dividend* and *divisor* so that it is set up for any example.



Lesson 15: Read Expressions in Which Letters Stand for Numbers

Classwork

Opening Exercise

Complete the graphic organizer with mathematical words that indicate each operation. Some words may indicate more than one operation.

<u>ADDITION</u> <u>SUBTRACTION</u> <u>MULTIPLICATION</u> <u>DIVISION</u> <u>EXPONENTS</u>

Example 1

Write an expression using words.

a. $a - b$

b. xy

c. $4f + p$

d. $d - b^3$

e. $5(u - 10) + h$

f. $\frac{3}{d+f}$

Exercises

Circle all the vocabulary words that could be used to describe the given expression.

1. $6h - 10$

ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION

2. $\frac{5d}{6}$

SUM	DIFFERENCE	PRODUCT	QUOTIENT

3. $5(2 + d) - 8$

ADD	SUBTRACT	MULTIPLY	DIVIDE

4. abc

MORE THAN	LESS THAN	TIMES	EACH

Write an expression using vocabulary to represent each given expression.

1. $8 - 2g$

2. $15(a + c)$

3. $\frac{m+n}{5}$

4. $b^3 - 18$

5. $f - \frac{d}{2}$

6. $\frac{u}{x}$

Lesson 16: Write Expressions in Which Letters Stand for Numbers

Classwork

Opening Exercise

Underline the key words in each statement and write an equivalent expression.

a. The sum of twice b and 5.

b. The quotient of c and d.

c. a raised to the fifth power then increased by the product of 5 and c.

d. The quantity of a plus b divided by 4.

e. 10 less than the product of 15 and c.

f. 5 times d then increased by 8.

Mathematical Modeling Exercise 2

Underline the text to show the key words. Write each real-world scenario as an expression using variables and numbers.

Marcus has 4 more dollars than Yaseen. If y is the amount of money Yaseen has, write an expression to show how much money Marcus has.

Mario is missing half of his assignments. If a represents the number of assignments, write an expression to show how many assignments Mario is missing.

Kamilah's weight has tripled since her first birthday. If w represents the amount Kamilah weighed on her first birthday, write an expression to show how much Kamilah weighs now.

Nathan brings cupcakes to school and gives them to his five best friends who share them equally. If c represents the number of cupcakes Nathan brings to school, write an expression to show how many cupcakes each of his friends receive.

Mrs. Marcus combines her atlases and dictionaries and then divides them among 10 different tables. If a represents the number of atlases and d represents the number of dictionaries Mrs. Marcus has, write an expression to show how many books would be on each table.

To improve in basketball, Ivan's coach told him that he needs to take four times as many free throws and four times as many jump shots every day. If f represents the number of free throws and j represents the number of jump shots Ivan shoots daily, write an expression to show how many shots he will need to take in order to improve in basketball.

Exercises

Mark the text by underlining key words, and then write an expression using variables and/or numbers for each statement.

1. b decreased by c squared.

2. 24 divided by the product of 2 and a.

3. 150 decreased by the quantity of 6 plus b.

4. The sum of twice c and 10.

5. Marlo had $\$35$ but then spent $\$m$.

6. Samantha saved her money and was able to quadruple the original amount, m.

7. Veronica increased her grade, g, by 4 points, and then doubled it.

8. Adbell had m pieces of candy and ate 5 of them. Then, he split the remaining candy equally among 4 friends.

9. To find out how much paint is needed, Mr. Jones must square the side length, s, of the gate, and then subtract 15.

10. Luis brought x cans of cola to the party, Faith brought d cans of cola, and De'Shawn brought h cans of cola. How many cans of cola did they bring altogether?

Lesson 17: Write Expressions in Which Letters Stand for Numbers

Classwork

Exercises

Station One	1. The sum of a and b.
	2. Five more than twice a number c.
	3. Martha bought d number of apples and then ate 6 of them.
Station Two	4. 14 decreased by p.
	5. The total of d and f, divided by 8.
	6. Rashod scored 6 less than 3 times as many baskets as Mike. Mike scores b baskets.
Station Three	7. The quotient of c and 6.
	8. Triple the sum of x and 17.
	9. Gabrielle had b buttons but then lost 6. Gabrielle took the remaining buttons and split them equally among her 5 friends.

Station Four	10. d doubled.
	11. Three more than 4 times a number x.
	12. Mali has c pieces of candy. She doubles the amount of candy she has then gives away 15 pieces.
Station Five	13. f cubed.
	14. The quantity of 4 increased by a, and then the sum is divided by 9.
	15. Tai earned 4 points fewer than double Oden's points. Oden earned p points.
Station Six	16. The difference between d and 8.
	17. 6 less than the sum of d and 9.
	18. Adalyn has x pants and s shirts. She combined them and sold half of them. How many items did Adalyn sell?

Lesson 18: Writing and Evaluating Expressions—Addition and Subtraction

Classwork

Opening Exercise

How can we show a number increased by 2?

Can you prove this using a model?

Example 1: The Importance of Being Specific in Naming Variables

When naming variables in expressions, it is important to be very clear about what they represent. The units of measure must be included if something is measured.

Exercises

1. Read the variable in the table, and improve the description given, making it more specific.

Variable	Incomplete Description	Complete Description with Units
Joshua's speed (J)	Let J = Joshua's speed	
Rufus's height (R)	Let R = Rufus's height	
Milk sold (M)	Let M = the amount of milk sold	
Colleen's time in the 40 meter hurdles (C)	Let C = Colleen's time	
Sean's age (S)	Let S = Sean's age	

2. Read each variable in the table and improve the description given, making it more specific.

Variable	Incomplete Description	Complete Description with Units
Karolyn's CDs (K)	Let K = Karolyn's CDs	Let K = the number of CDs Karolyn has
Joshua's merit badges (J)	Let J = Joshua's merit badges	
Rufus's trading cards (R)	Let R = Rufus's trading cards	
Milk money (M)	Let M = the amount of milk money	

Example 2: Writing and Evaluating Addition and Subtraction Expressions

Read each story problem. Identify the unknown quantity, and write an addition or subtraction expression that is described. Finally, evaluate your expression using the information given in column four.

Story Problem	Description with Units	Expression	Evaluate the Expression If:	Show Your Work and Evaluate
Gregg has two more dollars than his brother Jeff. Write an expression for the amount of money Gregg has.	Let j = Jeff's money in dollars	$j+2$	Jeff has $\$12$.	$j+2$ $12+2$ 14 Gregg has $\$14$.
Gregg has two more dollars than his brother Jeff. Write an expression for the amount of money Jeff has.	Let g = Gregg's money in dollars	$g-2$	Gregg has $\$14$.	$g-2$ $14-2$ 12 Jeff has $\$12$.

Abby read 8 more books than Kristen in the first marking period. Write an expression for the number of books Abby read.			Kristen read 9 books in the first marking period.	
Abby read 6 more books than Kristen in the second marking period. Write an expression for the number of books Kristen read.			Abby read 20 books in the second marking period.	
Daryl has been teaching for one year longer than Julie. Write an expression for the number of years that Daryl has been teaching.			Julie has been teaching for 28 years.	
Ian scored 4 fewer goals than Julia in the first half of the season. Write an expression for the number of goals Ian scored.			Julia scored 13 goals.	
Ian scored 3 fewer goals than Julia in the second half of the season. Write an expression for the number of goals Julia scored.			Ian scored 8 goals.	
Johann visited Niagara Falls 3 times fewer than Arthur. Write an expression for the number of times Johann visited Niagara Falls.			Arthur visited Niagara Falls 5 times.	

Lesson 19: Substituting to Evaluate Addition and Subtraction Expressions

Classwork

Opening Exercise

My older sister is exactly two years older than I am. Sharing a birthday is both fun and annoying. Every year on our birthday we have a party, which is fun, but she always brags that she is two years older than I am. Shown below is a table of our ages, starting when I was born:

My Age (in years)	My Sister's Age (in years)
0	2
1	3
2	4
3	5
4	6

a. Looking at the table, what patterns do you see? Tell a partner.

b. On the day I turned 8 years old, how old was my sister?

c. How do you know?

d. On the day I turned 16 years old, how old was my sister?

e. How do you know?

f. Do we need to extend the table to calculate these answers?

Example 1

My Age (in years)	My Sister's Age (in years)
0	2
1	3
2	4
3	5
4	6

a. What if you don't know how old I am? Let's use a variable for my age. Let $Y =$ my age in years. Can you develop an expression to describe how old my sister is?

b. Please add that to the last row of the table.

Example 2

My Age (in years)	My Sister's Age (in years)
0	2
1	3
2	4
3	5
4	6

a. How old was I when my sister was 6 years old?

b. How old was I when my sister was 15 years old?

c. How do you know?

d. Look at the table in Example 2. If you know my sister's age, can you determine my age?

e. If we use the variable G for my sister's age in years, what expression would describe my age in years?

f. Fill in the last row of the table with the expressions.

g. With a partner, calculate how old I was when my sister was 22, 23, and 24 years old.

Exercises

1. Noah and Carter are collecting box tops for their school. They each bring in 1 box top per day starting on the first day of school. However, Carter had a head start because his aunt sent him 15 box tops before school began. Noah's grandma saved 10 box tops, and Noah added those on his first day.

   a. Fill in the missing values that indicate the total number of box tops each boy brought to school.

School Day	Number of Box Tops Noah Has	Number of Box Tops Carter Has
1	11	16
2		
3		
4		
5		

   b. If we let D be the number of days since the new school year began, on day D of school, how many box tops will Noah have brought to school?

c. On day D of school, how many box tops will Carter have brought to school?

d. On day 10 of school, how many box tops will Noah have brought to school?

e. On day 10 of school, how many box tops will Carter have brought to school?

2. Shelly and Kristen share a birthday, but Shelly is 5 years older.

   a. Make a table showing their ages every year, beginning when Kristen was born.

b.	c.
d.	e.
f.	g.
h.	i.
j.	k.

   b. If Kristen is 16 years old, how old is Shelly?

   c. If Kristen is K years old, how old is Shelly?

   d. If Shelly is S years old, how old is Kristen?

Lesson 20: Writing and Evaluating Expressions—Multiplication and Division

Classwork

Example 1

1. The farmers' market is selling bags of apples. In every bag, there are 3 apples.

   a. Complete the table.

Number of Bags	Total Number of Apples
1	3
2	
3	
4	
B	

   b. What if the market had 25 bags of apples to sell? How many apples is that in all?

   c. If a truck arrived that had some number, a, more apples on it, then how many bags would the clerks use to bag up the apples?

   d. If a truck arrived that had 600 more apples on it, how many bags would the clerks use to bag up the apples?

   e. How is part (d) different from part (b)?

Exercises

2. In New York State, there is a five-cent deposit on all carbonated beverage cans and bottles. When you return the empty can or bottle, you get the five cents back.

   a. Complete the table.

Number of Containers Returned	Refund in Dollars
1	
2	
3	
4	
10	
50	
100	
C	

   b. If we let C represent the number of cans, what is the expression that shows how much money is returned?

   c. Use the expression to find out how much money Brett would receive if he returned 222 cans.

   d. If Gavin needs to earn $\$4.50$ for returning cans, how many cans does he need to collect and return?

   e. How is part (d) different from part (c)?

3. The fare for a subway or a local bus ride is $\$2.50$.

   a. Complete the table.

Number of Rides	Cost of Rides in Dollars
1	
2	
3	
4	
5	
10	
30	
R	

   b. If we let R represent the number of rides, what is the expression that shows the cost of the rides?

   c. Use the expression to find out how much money 60 rides would cost.

   d. If a commuter spends $\$175.00$ on subway or bus rides, how many trips did the commuter take?

   e. How is part (d) different from part (c)?

Challenge Problem

4. A pendulum swings though a certain number of cycles in a given time. Owen made a pendulum that swings 12 times every 15 seconds.

   a. Construct a table showing the number of cycles through which a pendulum swings. Include data for up to one minute. Use the last row for C cycles, and write an expression for the time it takes for the pendulum to make C cycles.

   b. Owen and his pendulum team set their pendulum in motion and counted 16 cycles. What was the elapsed time?

   c. Write an expression for the number of cycles a pendulum swings in S seconds.

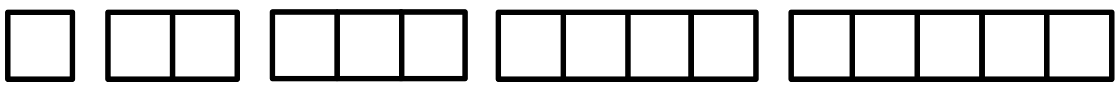
   d. In a different experiment, Owen and his pendulum team counted the cycles of the pendulum for 35 seconds. How many cycles did they count?

Lesson 21: Writing and Evaluating Expressions—Multiplication and Addition

Classwork

Mathematical Modeling Exercise

The Italian Villa Restaurant has square tables that the servers can push together to accommodate the customers. Only one chair fits along the side of the square table. Make a model of each situation to determine how many seats will fit around various rectangular tables.



Number of Square Tables	Number of Seats at the Table
1	
2	
3	
4	
5	
50	
200	
T	

Are there any other ways to think about solutions to this problem?

It is impractical to make a model of pushing 50 tables together to make a long rectangle. If we did have a rectangle that long, how many chairs would fit on the long sides of the table?

How many chairs fit on the ends of the long table?

How many chairs fit in all? Record it on your table.

Work with your group to determine how many chairs would fit around a very long rectangular table if 200 square tables were pushed together.

If we let T represent the number of square tables that make one long rectangular table, what is the expression for the number of chairs that will fit around it?

Example 1

Look at Example 1 with your group. Determine the cost for various numbers of pizzas, and also determine the expression that describes the cost of having P pizzas delivered.

a. Pizza Queen has a special offer on lunch pizzas: $\$4.00$ each. They charge $\$2.00$ to deliver, regardless of how many pizzas are ordered. Determine the cost for various numbers of pizzas, and also determine the expression that describes the cost of having P pizzas delivered.

Number of Pizzas Delivered	Total Cost in Dollars
1	
2	
3	
4	
10	
50	
P	

What mathematical operations did you need to perform to find the total cost?

Suppose our principal wanted to buy a pizza for everyone in our class? Determine how much this would cost.

b. If the booster club had $\$400$ to spend on pizza, what is the greatest number of pizzas they could order?

c. If the pizza price was raised to $\$5.00$ and the delivery price was raised to $\$3.00$, create a table that shows the total cost (pizza plus delivery) of 1, 2, 3, 4, and 5 pizzas. Include the expression that describes the new cost of ordering P pizzas.

Number of Pizzas Delivered	**Total Cost in Dollars**
1	
2	
3	
4	
5	
P	

Lesson 23: True and False Number Sentences

Classwork

Opening Exercise

Determine what each symbol stands for and provide an example.

Symbol	What the Symbol Stands For	Example
$=$		
$>$		
$<$		
$\leq$		
$\geq$		

Example 1

For each equation or inequality your teacher displays, write the equation or inequality, and then substitute 3 for every x. Determine if the equation or inequality results in a true number sentence or a false number sentence.

Exercises

Substitute the indicated value into the variable, and state (in a complete sentence) whether the resulting number sentence is true or false. If true, find a value that would result in a false number sentence. If false, find a value that would result in a true number sentence.

1. $4 + x = 12$. Substitute 8 for x.

2. $3g > 15$. Substitute $4\frac{1}{2}$ for g.

3. $\frac{f}{4} < 2$. Substitute 8 for f.

4. $14.2 \leq h - 10.3$. Substitute 25.8 for h.

5. $4 = \frac{8}{h}$. Substitute 6 for h.

6. $3 > k + \frac{1}{4}$. Substitute $1\frac{1}{2}$ for k.

7. $4.5 - d > 2.5$. Substitute 2.5 for d.

8. $8 \geq 32p$. Substitute $\frac{1}{2}$ for p.

9. $\frac{w}{2} < 32$. Substitute 16 for w.

10. $18 \leq 32 - b$. Substitute 14 for b.

Lesson 24: True and False Number Sentences

Classwork

Opening Exercise

State whether each number sentence is true or false. If the number sentence is false, explain why.

a. $4 + 5 > 9$

b. $3 \cdot 6 = 18$

c. $32 > \frac{64}{4}$

d. $78 - 15 < 68$

e. $22 \geq 11 + 12$

Example 1

Write true or false if the number substituted for g results in a true or false number sentence.

Substitute g with	$4g = 32$	$g = 8$	$3g \geq 30$	$g \geq 10$	$\frac{g}{2} > 2$	$g > 4$	$30 \geq 38 - g$	$g \geq 8$
8								
4								
2								
0								
10								

Example 2

State when the following equations/inequalities will be true and when they will be false.

a. $r + 15 = 25$

b. $6 - d > 0$

c. $\frac{1}{2}f = 15$

d. $\frac{y}{3} < 10$

e. $7g \geq 42$

f. $a - 8 \leq 15$

Exercises

Complete the following problems with a partner. State when the following equations and inequalities will be true and when they will be false.

1. $15c > 45$

2. $25 = d - 10$

3. $56 \geq 2e$

4. $\frac{h}{5} \geq 12$

5. $45 > h + 29$

6. $4a \leq 16$

7. $3x = 24$

Identify all equality and inequality signs that can be placed into the blank to make a true number sentence.

8. $15 + 9$ ______ 24

9. $8 \cdot 7$ ______ 50

10. $\frac{15}{2}$ ______ 10

11. 34 ______ $17 \cdot 2$

12. 18 ______ $24.5 - 6$

Lesson 25: Finding Solutions to Make Equations True

Classwork

Opening Exercise

Identify a value for the variable that would make each equation or inequality into a true number sentence. Is this the only possible answer? State when the equation or inequality is true using equality and inequality symbols.

a. $3 + g = 15$

b. $30 > 2d$

c. $\frac{15}{f} < 5$

d. $42 \leq 50 - m$

Example 1

Each of the following numbers, if substituted for the variable, makes one of the equations below into a true number sentence. Match the number to that equation: 3, 6, 15, 16, 44.

a. $n + 26 = 32$

b. $n - 12 = 32$

c. $17n = 51$

d. $4^2 = n$

e. $\frac{n}{3} = 5$

Lesson 26: One-Step Equations—Addition and Subtraction

Classwork

Exercise 1

Solve each equation. Use both tape diagrams and algebraic methods for each problem. Use substitution to check your answers.

a. $b + 9 = 15$

b. $12 = 8 + c$

Exercise 2

Given the equation $d - 5 = 7$:

a. Demonstrate how to solve the equation using tape diagrams.

b. Demonstrate how to solve the equation algebraically.

c. Check your answer.

Exercise 3

Solve each problem, and show your work. You may choose which method (tape diagrams or algebraically) you prefer. Check your answers after solving each problem.

a. $e + 12 = 20$

b. $f - 10 = 15$

c. $g - 8 = 9$

Lesson 27: One-Step Equations—Multiplication and Division

Classwork

Example 1

Solve $3z = 9$ using tape diagrams and algebraically. Then, check your answer.

First, draw two tape diagrams, one to represent each side of the equation.

If 9 had to be split into three groups, how big would each group be?

Demonstrate the value of z using tape diagrams.

How can we demonstrate this algebraically?

How does this get us the value of z?

How can we check our answer?

Example 2

Solve $\frac{y}{4} = 2$ using tape diagrams and algebraically. Then, check your answer.

First, draw two tape diagrams, one to represent each side of the equation.

If the first tape diagram shows the size of $y \div 4$, how can we draw a tape diagram to represent y?

Draw this tape diagram.

What value does each $y \div 4$ section represent? How do you know?

How can you use a tape diagram to show the value of y?

How can we demonstrate this algebraically?

How does this help us find the value of y?

How can we check our answer?

Exercises

1. Use tape diagrams to solve the following problem: $3m = 21$.

2. Solve the following problem algebraically: $15 = \frac{n}{5}$.

3. Calculate the solution of the equation using the method of your choice: $4p = 36$.

4. Examine the tape diagram below, and write an equation it represents. Then, calculate the solution to the equation using the method of your choice.

70

q	q	q	q	q	q	q

5. Write a multiplication equation that has a solution of 12. Use tape diagrams to prove that your equation has a solution of 12.

6. Write a division equation that has a solution of 12. Prove that your equation has a solution of 12 using algebraic methods.

Lesson 31: Problems in Mathematical Terms

Classwork

Example 1

Marcus reads for 30 minutes each night. He wants to determine the total number of minutes he will read over the course of a month. He wrote the equation $t = 30d$ to represent the total amount of time that he has spent reading, where t represents the total number of minutes read and d represents the number of days that he read during the month. Determine which variable is independent and which is dependent. Then, create a table to show how many minutes he has read in the first seven days.

Independent variable ____________________

Dependent variable ____________________

Example 2

Kira designs websites. She can create three different websites each week. Kira wants to create an equation that will give her the total number of websites she can design given the number of weeks she works. Determine the independent and dependent variables. Create a table to show the number of websites she can design over the first 5 weeks. Finally, write an equation to represent the number of websites she can design when given any number of weeks.

Independent variable ______________________________

Dependent variable ______________________________

Equation ______________________________

Example 3

Priya streams movies through a company that charges her a $5 monthly fee plus $1.50 per movie. Determine the independent and dependent variables, write an equation to model the situation, and create a table to show the total cost per month given that she might stream between 4 and 10 movies in a month.

Independent variable ______________________________

Dependent variable ______________________________

Equation ______________________________

Exercises 1–4

1. Sarah is purchasing pencils to share. Each package has 12 pencils. The equation $n = 12p$, where n is the total number of pencils and p is the number of packages, can be used to determine the total number of pencils Sarah purchased. Determine which variable is dependent and which is independent. Then, make a table showing the number of pencils purchased for 3–7 packages.

2. Charlotte reads 4 books each week. Let b be the number of books she reads each week, and let w be the number of weeks that she reads. Determine which variable is dependent and which is independent. Then, write an equation to model the situation, and make a table that shows the number of books read in under 6 weeks.

3. A miniature golf course has a special group rate. You can pay $\$20$ plus $\$3$ per person when you have a group of 5 or more friends. Let f be the number of friends and c be the total cost. Determine which variable is independent and which is dependent, and write an equation that models the situation. Then, make a table to show the cost for 5 to 12 friends.

4. Carlos is shopping for school supplies. He bought a pencil box for $\$3$, and he also needs to buy notebooks. Each notebook is $\$2$. Let t represent the total cost of the supplies and n be the number of notebooks Carlos buys. Determine which variable is independent and which is dependent, and write an equation that models the situation. Then, make a table to show the cost for 1 to 5 notebooks.

Lesson 32: Multi-Step Problems in the Real World

Classwork

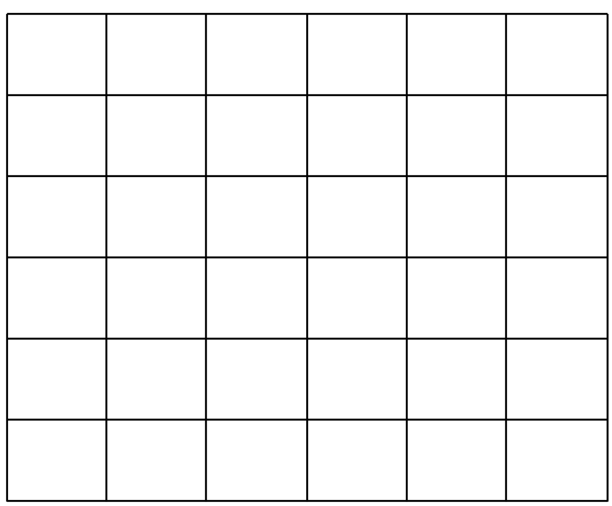
Opening Exercise

Xin is buying beverages for a party that come in packs of 8. Let p be the number of packages Xin buys and t be the total number of beverages. The equation $t = 8p$ can be used to calculate the total number of beverages when the number of packages is known. Determine the independent and dependent variable in this scenario. Then, make a table using whole number values of p less than 6.

Number of Packages (p)	Total Number of Beverages ($t = 8p$)
0	
1	
2	
3	
4	
5	

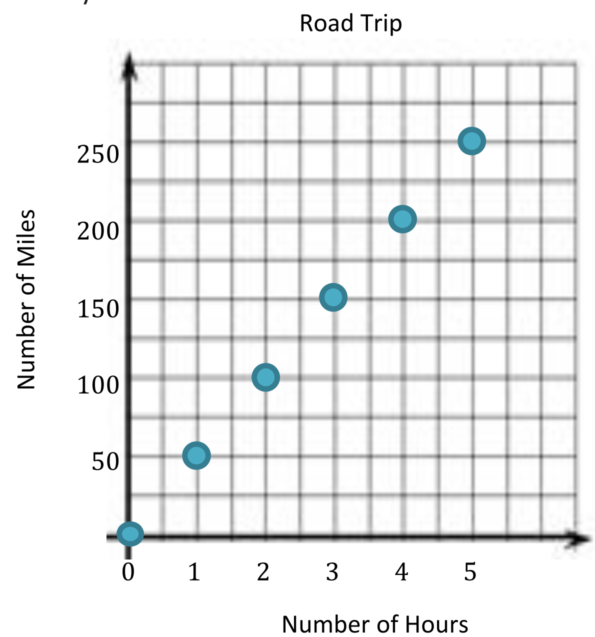
Example 1

Make a graph for the table in the Opening Exercise.



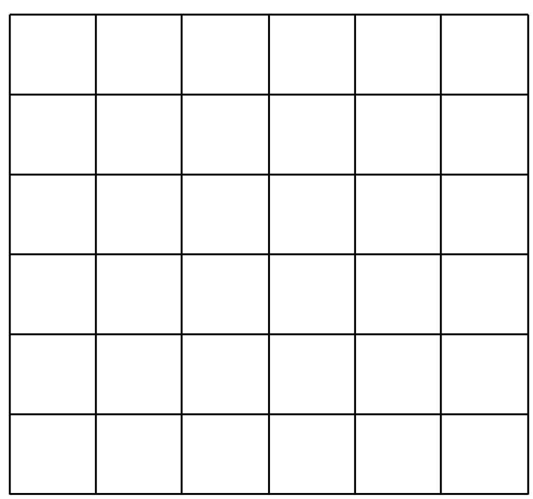
Example 2

Use the graph to determine which variable is the independent variable and which is the dependent variable. Then, state the relationship between the quantities represented by the variables.



Exercises

1. Each week Quentin earns $\$30$. If he saves this money, create a graph that shows the total amount of money Quentin has saved from week 1 through week 8. Write an equation that represents the relationship between the number of weeks that Quentin has saved his money, w, and the total amount of money in dollars that he has saved, s. Then, name the independent and dependent variables. Write a sentence that shows this relationship.



2. Zoe is collecting books to donate. She started with 3 books and collects two more each week. She is using the equation $b = 2w + 3$, where b is the total number of books collected and w is the number of weeks she has been collecting books. Name the independent and dependent variables. Then, create a graph to represent how many books Zoe has collected when w is 5 or less.

3. Eliana plans to visit the fair. She must pay \$5 to enter the fair grounds and an additional \$3 per ride. Write an equation to show the relationship between r, the number of rides, and t, the total cost. State which variable is dependent and which is independent. Then, create a graph that models the equation.

Lesson 33: From Equations to Inequalities

Classwork

Example 1

What value(s) does the variable have to represent for the equation or inequality to result in a true number sentence? What value(s) does the variable have to represent for the equation or inequality to result in a false number sentence?

a. $y + 6 = 16$

b. $y + 6 > 16$

c. $y + 6 \geq 16$

d. $3g = 15$

e. $3g < 15$

f. $3g \leq 15$

Example 2

Which of the following number(s), if any, make the equation or inequality true: $\{0, 3, 5, 8, 10, 14\}$?

a. $m + 4 = 12$

b. $m + 4 < 12$

c. $f - 4 = 2$

d. $f - 4 > 2$

e. $\frac{1}{2}h = 8$

f. $\frac{1}{2}h \geq 8$

Exercises 1–8

Choose the number(s), if any, that make the equation or inequality true from the following set of numbers: $\{0, 1, 5, 8, 11, 17\}$.

1. $m + 5 = 6$

2. $m + 5 \leq 6$

3. $5h = 40$

4. $5h > 40$

5. $\frac{1}{2}y = 5$

6. $\frac{1}{2}y \leq 5$

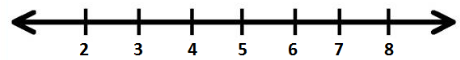
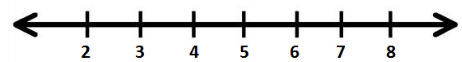
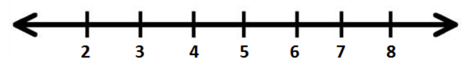
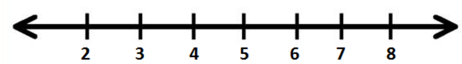
7. $k - 3 = 20$

8. $k - 3 > 20$

Lesson 34: Writing and Graphing Inequalities in Real-World Problems

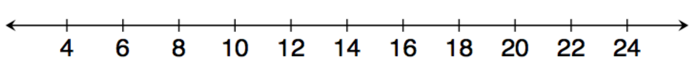
Classwork

Example 1

Statement	Inequality	Graph
a. Caleb has at least $5.	__________	
b. Tarek has more than $5.	__________	
c. Vanessa has at most $5.	__________	
d. Li Chen has less than $5.	__________	

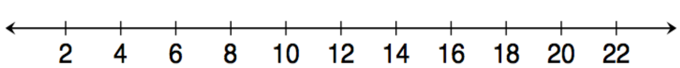
Example 2

Kelly works for Quick Oil Change. If customers have to wait longer than 20 minutes for the oil change, the company does not charge for the service. The fastest oil change that Kelly has ever done took 6 minutes. Show the possible customer wait times in which the company charges the customer.



Example 3

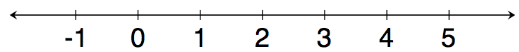
Gurnaz has been mowing lawns to save money for a concert. Gurnaz will need to work for at least six hours to save enough money, but he must work fewer than 16 hours this week. Write an inequality to represent this situation, and then graph the solution.



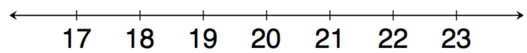
Exercises 1–5

Write an inequality to represent each situation. Then, graph the solution.

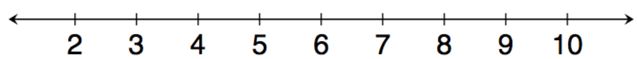
1. Blayton is at most 2 meters above sea level.



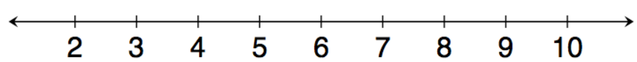
2. Edith must read for a minimum of 20 minutes.



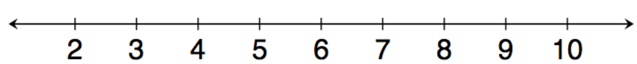
3. Travis milks his cows each morning. He has never gotten fewer than 3 gallons of milk; however, he always gets fewer than 9 gallons of milk.



4. Rita can make 8 cakes for a bakery each day. So far she has orders for more than 32 cakes. Right now, Rita needs more than four days to make all 32 cakes.

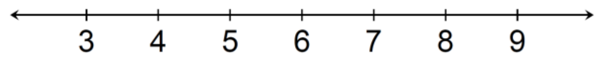


5. Rita must have all the orders placed right now done in 7 days or fewer. How will this change your inequality and your graph?

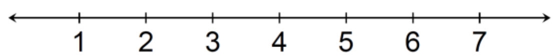


Possible Extension Exercises 6–10

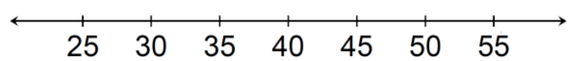
6. Kasey has been mowing lawns to save up money for a concert. He earns $\$15$ per hour and needs at least $\$90$ to go to the concert. How many hours should he mow?



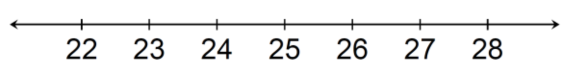
7. Rachel can make 8 cakes for a bakery each day. So far, she has orders for more than 32 cakes. How many days will it take her to complete the orders?



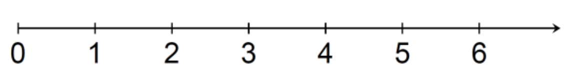
8. Ranger saves $\$70$ each week. He needs to save at least $\$2{,}800$ to go on a trip to Europe. How many weeks will he need to save?



9. Clara has less than $\$75$. She wants to buy 3 pairs of shoes. What price shoes can Clara afford if all the shoes are the same price?

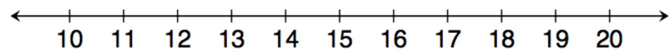


10. A gym charges $\$25$ per month plus $\$4$ extra to swim in the pool for an hour. If a member only has $\$45$ to spend each month, at most how many hours can the member swim?

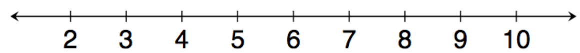


Write and graph an inequality for each problem.

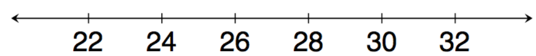
11. At least 13.



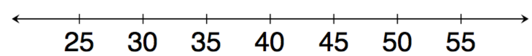
12. Less than 7.



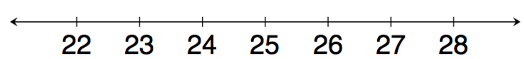
13. Chad will need at least 24 minutes to complete the 5K race. However, he wants to finish in under 30 minutes.



14. Eva saves \$60 each week. Since she needs to save at least \$2,400 to go on a trip to Europe, she will need to save for at least 40 weeks.



15. Clara has \$100. She wants to buy 4 pairs of the same pants. Due to tax, Clara can afford pants that are less than \$25.



16. A gym charges \$30 per month plus \$4 extra to swim in the pool for an hour. Because a member has just \$50 to spend at the gym each month, the member can swim at most 5 hours.